

Testing Explanations of the $B \rightarrow \phi K^*$ Polarization Puzzle

Alakabha Datta ^{a,1}, Andrei V. Gritsan ^{b,2}, David London ^{c,3},
Makiko Nagashima ^{c,4}, and Alejandro Szynkman ^{c,5}

*a: Dept. of Physics and Astronomy, 108 Lewis Hall,
University of Mississippi, Oxford, MS 38677-1848, USA*
*b: Dept. of Physics and Astronomy, Johns Hopkins University,
Baltimore, MD 21218, USA*
*c: Physique des Particules, Université de Montréal,
C.P. 6128, succ. centre-ville, Montréal, QC, Canada H3C 3J7*

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Abstract

$B \rightarrow \phi K^*$ ($\bar{b} \rightarrow \bar{s}$) is three separate decays, one for each polarization of the final-state vector mesons (one longitudinal, two transverse). It is observed that the fraction of transverse decays, f_T , and the fraction of longitudinal decays, f_L , are roughly equal: $f_T/f_L \simeq 1$, in opposition to the naive expectation that $f_T \ll f_L$. If one requires a single explanation of all polarization puzzles, two possibilities remain within the standard model: penguin annihilation and rescattering. In this paper we examine the predictions of these two explanations for f_T/f_L in $\bar{b} \rightarrow \bar{d}$ decays. In $B \rightarrow \rho\rho$ decays, only $B_d^0 \rightarrow \rho^0\rho^0$ can possibly exhibit a large f_T/f_L . In B decays related by U-spin, we find two promising possibilities: (i) $B^+ \rightarrow K^{*0}\rho^+$ ($\bar{b} \rightarrow \bar{s}$) and $B^+ \rightarrow \bar{K}^{*0}K^{*+}$ ($\bar{b} \rightarrow \bar{d}$) and (ii) $B_s \rightarrow K^{*0}\bar{K}^{*0}$ ($\bar{b} \rightarrow \bar{s}$) and $B_d^0 \rightarrow \bar{K}^{*0}K^{*0}$ ($\bar{b} \rightarrow \bar{d}$). The measurement of f_T/f_L in these pairs of decays will allow us to test penguin annihilation and rescattering. Finally, it is possible to distinguish penguin annihilation from rescattering by performing a time-dependent angular analysis of $B_d^0 \rightarrow \bar{K}^{*0}K^{*0}$.

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¹datta@phy.olemiss.edu

²gritsan@jhu.edu

³london@lps.umontreal.ca

⁴makiko@lps.umontreal.ca

⁵szynkman@lps.umontreal.ca

1 Introduction

The B -factories BABAR and Belle, along with Tevatron experiments, have been operating for several years now, making many measurements of various B decays. As always, the hope is to find results which are in contradiction with the expectations of the standard model (SM) and which therefore show evidence for the presence of physics beyond the SM. To date, there have been several hints of such new physics in $\bar{b} \rightarrow \bar{s}$ transitions, though none has been statistically significant.

One intriguing puzzle was first seen in $B \rightarrow \phi K^*$ decays [1]. In this decay the final-state particles are vector mesons. Thus, when the spin of the vector mesons is taken into account, this decay is in fact three separate decays, one for each polarization (one longitudinal, two transverse). Naively, the transverse amplitudes are suppressed by a factor of size m_V/m_B (V is one of the vector mesons) with respect to the longitudinal amplitude. As such, one expects the fraction of transverse decays, f_T , to be much less than the fraction of longitudinal decays, f_L . However, it is observed that these two fractions are roughly equal: $f_T/f_L(B \rightarrow \phi K^*) \simeq 1$ (see Table 1).

Mode	$\mathcal{B}(10^{-6})$	f_L	f_\perp	$\phi_\parallel - \pi$	$\phi_\perp - \pi$
ϕK^{*0} [4, 5, 6]	9.5 ± 0.9	0.49 ± 0.04	$0.27^{+0.04}_{-0.03}$	$-0.73^{+0.18}_{-0.16}$	-0.62 ± 0.17
ϕK^{*+} [1, 5, 7]	10.0 ± 1.1	0.50 ± 0.05	0.20 ± 0.05	-0.80 ± 0.17	-0.56 ± 0.17
$\rho^+ K^{*0}$ [8, 9]	9.2 ± 1.5	0.48 ± 0.08	—	—	—
$\rho^0 K^{*0}$ [9]	5.6 ± 1.6	0.57 ± 0.12	—	—	—
$\rho^- K^{*+}$ [9]	< 12.0	—	—	—	—
$\rho^0 K^{*+}$ [9]	$(3.6^{+1.9}_{-1.8})$	(0.9 ± 0.2)	—	—	—
ωK^{*0} [10]	(2.4 ± 1.3)	$(0.71^{+0.27}_{-0.24})$	—	—	—
ωK^{*+} [10]	< 3.4	—	—	—	—

Table 1: Measurements of the branching fraction \mathcal{B} , longitudinal polarization fraction f_L , fraction of parity-odd transverse amplitude f_\perp , and phases of the two transverse amplitudes ϕ_\parallel and ϕ_\perp (rad) with respect to the longitudinal amplitude, for $B \rightarrow \phi K^*$, ρK^* , and ωK^* , expected to proceed through a $\bar{b} \rightarrow \bar{s}$ transition [2, 3]. Numbers in parentheses indicate observables measured with less than 4σ significance. We quote the solution of ϕ_\parallel and ϕ_\perp according to the phase ambiguity resolved by BABAR [4, 7]. For a complete list of up to 12 parameters measured, including CP-violating observables, see references quoted.

Within the SM, there are three potential explanations of the observed f_T/f_L ratio, described in more detail in Sec. 2: penguin annihilation [11], rescattering [12, 13], and enhanced penguin contributions due to the dipole operator [14, 15]. Assuming only one of these explanations is valid, enhanced dipole-operator contributions are

ruled out by the observed large f_T/f_L in $B^+ \rightarrow \rho^+ K^{*0}$ (see Table 1). However, the other two are in agreement with all observed data. In this paper, we explore ways of testing these explanations.

These two explanations account for a large f_T/f_L in $\bar{b} \rightarrow \bar{s}$ decays. However, the key point is that a large f_T/f_L is also predicted in certain $\bar{b} \rightarrow \bar{d}$ decays [16]. We examine these predictions. The measurement of f_T/f_L in these $\bar{b} \rightarrow \bar{d}$ decays will allow us to test penguin annihilation and rescattering as the explanations of the observed f_T/f_L ratio in $B \rightarrow \phi K^*$ decays, or maybe even rule them out.

In Sec. 2, we describe in more detail the SM explanations of the observed f_T/f_L ratio, along with constraints from present data. Sec. 3 contains the predictions for f_T/f_L in $B \rightarrow \rho\rho$ decays. Only $B_d^0 \rightarrow \rho^0\rho^0$ is expected to possibly exhibit a large f_T/f_L . In Sec. 4, we examine f_T/f_L for various B decays related by U-spin. Sec. 5 contains a method for distinguishing penguin annihilation from rescattering. We conclude in Sec. 6.

2 Explanations of f_T/f_L in $B \rightarrow \phi K^*$

We focus here on $B \rightarrow V_1 V_2$ decays (V_i is a vector meson). This is really three decays, one for each polarization of the final state. Here it is useful to use the linear polarization basis, where one decomposes the decay amplitude into components in which the polarizations of the final-state vector mesons (ε_i^*) are either longitudinal (A_0), or transverse to their directions of motion and parallel (A_{\parallel}) or perpendicular (A_{\perp}) to one another. The amplitude for this decay is given by [17, 18]

$$M = A_0 \varepsilon_1^{*L} \cdot \varepsilon_2^{*L} - \frac{1}{\sqrt{2}} A_{\parallel} \bar{\varepsilon}_1^{*T} \cdot \bar{\varepsilon}_2^{*T} - \frac{i}{\sqrt{2}} A_{\perp} \bar{\varepsilon}_1^{*T} \times \bar{\varepsilon}_2^{*T} \cdot \hat{p} , \quad (1)$$

where \hat{p} is the unit vector along the direction of motion of V_2 in the rest frame of V_1 , $\varepsilon_i^{*L} = \bar{\varepsilon}_i^* \cdot \hat{p}$, and $\bar{\varepsilon}_i^{*T} = \bar{\varepsilon}_i^* - \varepsilon_i^{*L} \hat{p}$. In this paper we will often use the basis A_{\pm} for the transverse polarizations, where $A_{\pm} = (A_{\parallel} \pm A_{\perp})/\sqrt{2}$. Note that, due to the factor of ‘ i ’ in the amplitude above, A_+ and A_- change roles in the CP-conjugate decay $\bar{B} \rightarrow \bar{V}_1 \bar{V}_2$: $A_+ \rightarrow \bar{A}_-$ and $A_- \rightarrow \bar{A}_+$.

The fraction of various types of decay is given by

$$f_L = \frac{|A_0|^2}{|A_0|^2 + |A_+|^2 + |A_-|^2} , \quad f_T = \frac{|A_+|^2 + |A_-|^2}{|A_0|^2 + |A_+|^2 + |A_-|^2} , \quad (2)$$

where $f_T = (1 - f_L)$,

$$f_{\perp} = \frac{|A_{\perp}|^2}{|A_0|^2 + |A_+|^2 + |A_-|^2} , \quad f_{\parallel} = \frac{|A_{\parallel}|^2}{|A_0|^2 + |A_+|^2 + |A_-|^2} , \quad (3)$$

where $f_{\parallel} = (1 - f_L - f_{\perp})$, and the relative phases are

$$\phi_{\perp} = \arg(A_{\perp}/A_0) , \quad \phi_{\parallel} = \arg(A_{\parallel}/A_0) . \quad (4)$$

We note that when $\phi_\perp = \phi_\parallel$ and $f_\perp = f_\parallel$, we have $A_- = 0$, which is close to experimental observation for $B \rightarrow \phi K^*$ in Table 1.

In the introduction we noted that there are three SM explanations of the observed f_T/f_L in $B \rightarrow \phi K^*$ decays. We discuss them in more detail here.

We begin with penguin annihilation [11], as shown in Fig. 1. $B \rightarrow \phi K^*$ receives penguin contributions, $\bar{b}\mathcal{O}s\bar{q}\mathcal{O}q$, where $q = u, d$ (\mathcal{O} are Lorentz structures, and color indices are suppressed). Applying a Fierz transformation, these operators can be written as $\bar{b}\mathcal{O}'q\bar{q}\mathcal{O}'s$. A gluon can now be emitted from one of the quarks in the operators which can then produce a pair of s, \bar{s} quarks. These then combine with the \bar{s}, q quarks to form the final states ϕK^{*+} ($q = u$) or ϕK^{*0} ($q = d$).

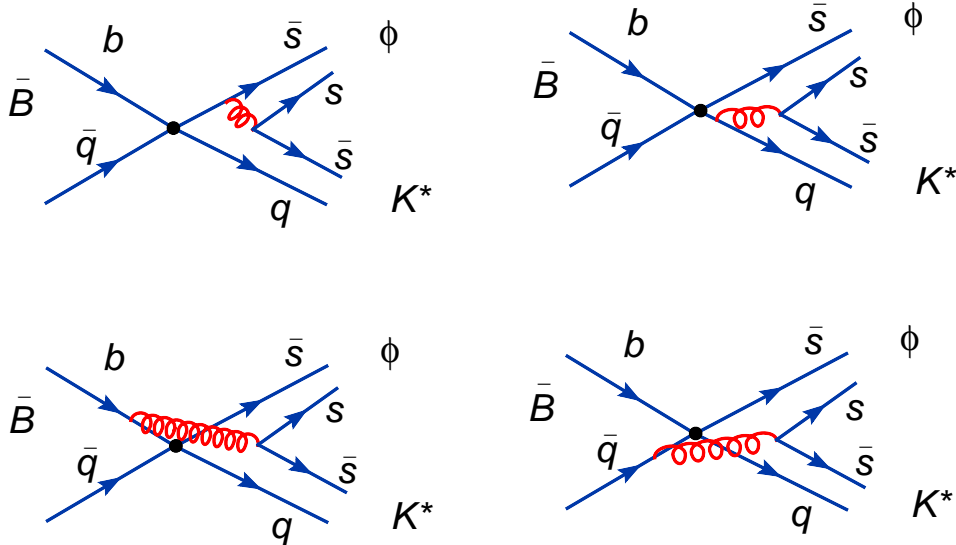


Figure 1: The penguin annihilation diagrams.

Normally all annihilation contributions are expected to be small as they are higher order in the $1/m_b$ expansion, and thus ignored. However, within QCD factorization (QCdf) [19], it is plausible that the coefficients of these terms are large [11]. In QCdf penguin annihilation is not calculable because of divergences which are parameterized in terms of unknown quantities. One may choose these parameters to fit the polarization data in $B \rightarrow \phi K^*$ decays. (Within perturbative QCD [20], the penguin annihilation is calculable and can be large, though it is not large enough to explain the polarization data in $B \rightarrow \phi K^*$ [21].) Note that the penguin annihilation term arises only from penguin operators with an internal t quark.

We now turn to rescattering [12, 13], shown in Fig. 2. It has been suggested that rescattering effects involving charm intermediate states, generated by the operator $\bar{b}\mathcal{O}'c\bar{c}\mathcal{O}'s$, can produce large transverse polarization in $B \rightarrow \phi K^*$. A particular

realization of this scenario is the following [12]. Consider the decay $B^+ \rightarrow D_s^{*+} \bar{D}^{*0}$ generated by the operator $\bar{b}\mathcal{O}'c\bar{c}\mathcal{O}'s$. Since the final-state vector mesons are heavy, the transverse polarization can be large. The state $D_s^{*+} \bar{D}^{*0}$ can now rescatter to ϕK^{*+} . If the transverse polarization T is not reduced in the scattering process, this will lead to $B^+ \rightarrow \phi K^{*+}$ with large f_T/f_L . (A similar rescattering effect can take place for $B_d^0 \rightarrow \phi K^{*0}$.)

In principle, rescattering can also take place if $u\bar{u}$ quark pairs are involved. However, this does not contribute significantly to T . One way to see this is to realize that most intermediate states are light, so that the transverse polarization is small. Thus, one cannot obtain a large f_T/f_L in this case.

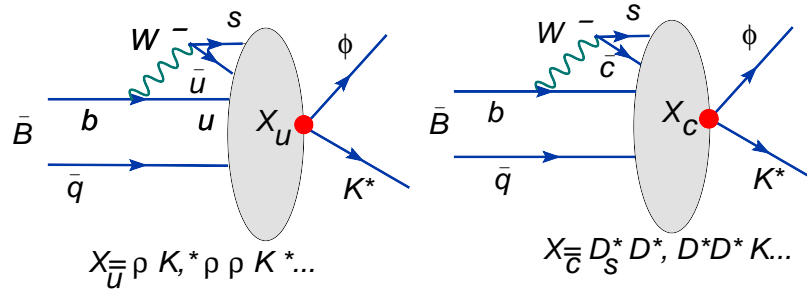


Figure 2: The rescattering diagrams.

Finally, we examine electroweak-penguin (EWP) contributions to $B \rightarrow \phi K^*$. The standard EWP diagrams contribute mainly to f_L . However, in Ref. [14], it was pointed out that electromagnetic effects involving a photon that subsequently converts to a vector meson can generate an unsuppressed transverse amplitude. (Enhanced chromomagnetic dipole operators are discussed in Ref. [15], with similar results as Ref. [14].) With this new EWP mechanism, the observed value of f_T/f_L in $B \rightarrow \phi K^*$ may be explained, but it requires that this dipole EWP contribution which enhances one of the transverse amplitudes by $\alpha_{em} m_b / \Lambda_{QCD}$ be sufficiently strong [14]. Note that this electromagnetic contribution, and hence a large value of f_T/f_L , should be observed in any decay where the photon can convert into a neutral vector meson. However, a large f_T/f_L is observed in $B^+ \rightarrow \rho^+ K^{*0}$ decays (see Table 1), but no EWP can contribute here. Thus, the new enhanced EWP's cannot be the sole explanation of a large f_T/f_L . On the other hand, in this paper we assume that there is a single explanation for the large f_T/f_L 's, and so enhanced EWP contributions of the nature discussed above are ruled out.

There are therefore only two proposed SM explanations of the observed f_T/f_L in $B \rightarrow \phi K^*$ decays: penguin annihilation and rescattering. At this point, it is useful to make a general comment about the two explanations. Penguin annihilation holds within a specific calculation framework (QCdf). However, rescattering is just a scenario – there isn't even a concrete model. One can come up with a particular

model to implement rescattering [12], but if it fails, it doesn't rule out the idea – one can simply invent other models.

The naive expectation of small f_T/f_L can be extended to the hierarchy $|A_0|^2 \gg |A_+|^2 \gg |A_-|^2$. While both penguin annihilation and rescattering ideas were proposed to explain the violation of $|A_0|^2 \gg |A_\pm|^2$, the inequality $|A_+|^2 \gg |A_-|^2$ may also be used to test the models. As we noted earlier, the experimental observation of Table 1 is indeed consistent with $|A_+|^2 \gg |A_-|^2$. Simple models of rescattering [12] violate this inequality, which is not supported by experimental data. While this does not rule out the rescattering idea, this makes it a less likely explanation. On the other hand, penguin annihilation idea is consistent with $|A_+|^2 \gg |A_-|^2$.

Although the physical origin of penguin annihilation and rescattering is different, the two explanations have similarities of calculation. In order to see this, consider the penguin contribution \mathcal{P}_q for the decay $\bar{b} \rightarrow \bar{q}q'q'$ ($q = d, s, q' = u, d, s$):

$$\begin{aligned} \mathcal{P}_q &= V_{ub}^* V_{uq} P_u + V_{cb}^* V_{cq} P_c + V_{tb}^* V_{tq} P_t \\ &= V_{cb}^* V_{cq} (P_c - P_u) + V_{tb}^* V_{tq} (P_t - P_u) , \end{aligned} \quad (5)$$

where the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix has been used in the second line. In the rescattering solution the dominant contribution to the transverse amplitudes come from P_c , while the contributions from $P_{u,t}$ are small. In the penguin annihilation solution the dominant contributions to the transverse amplitudes come from P_t through the penguin annihilation diagram, and the contributions from $P_{u,c}$ are small. Thus, in either case, the effect of the dominant contribution to the transverse amplitudes is simply the addition of one amplitude. Below we follow this prescription: we take into account the additional SM effects by adding a single amplitude to represent the dominant contribution to the transverse amplitudes.

3 $B \rightarrow \rho\rho$ Decays

Both penguin annihilation and rescattering explain the f_T/f_L ratio in the $\bar{b} \rightarrow \bar{s}$ decay $B \rightarrow \phi K^*$ by modifying the penguin amplitude. A similar modification must appear in some $\bar{b} \rightarrow \bar{d}$ decays. In this section we examine the predictions of these explanations for f_T/f_L in $B \rightarrow \rho\rho$ decays. Experimental measurements in $B \rightarrow \rho\rho$ along with related $B \rightarrow \rho\omega, \omega\omega$, and $K^* \bar{K}^*$ decays are shown in Table 2. However, due to additional uncertainties, we do not consider modes with ω further in this paper. We will discuss $K^* \bar{K}^*$ decays in the next section.

Within the diagrammatic approach [26], the three $B \rightarrow \rho\rho$ amplitudes are given mainly by three diagrams: the color-favored and color-suppressed tree amplitudes Tr and C , and the gluonic penguin amplitude P .

$$-\sqrt{2}A(B^+ \rightarrow \rho^+ \rho^0) = Tr + C ,$$

Mode	$\mathcal{B}(10^{-6})$	f_L	Tr	C	P
$\rho^0 \rho^+$ [22]	18.2 ± 3.0	$0.912^{+0.044}_{-0.045}$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	0
$\rho^+ \rho^-$ [23]	$24.2^{+3.0}_{-3.2}$	$0.976^{+0.028}_{-0.024}$	-1	0	-1
$\rho^0 \rho^0$ [24]	(1.07 ± 0.38)	$(0.86^{+0.12}_{-0.14})$	0	$-1/\sqrt{2}$	$1/\sqrt{2}$
$\omega \rho^+$ [10]	$10.6^{+2.6}_{-2.3}$	0.88 ± 0.11	$1/\sqrt{2}$	$1/\sqrt{2}$	$\sqrt{2}$
$\omega \rho^0$ [10]	< 1.5	—	0	0	1
$\omega \omega$ [10]	< 4.0	—	0	$1/\sqrt{2}$	$1/\sqrt{2}$
$K^{*0} \bar{K}^{*0}$ [25]	< 22	—	0	0	1
$K^{*+} \bar{K}^{*0}$ [25]	< 71	—	0	0	1

Table 2: Measurements of the branching fraction \mathcal{B} and longitudinal polarization fraction f_L for B^+ and B_d^0 meson decays expected to proceed through the $\bar{b} \rightarrow \bar{d}$ transition [2, 3]. Numbers in parentheses indicate observables measured with less than 4σ significance. The last three columns show the naive amplitude decomposition in terms of color-favored and color-suppressed tree amplitudes Tr and C and the gluonic penguin amplitude P .

$$\begin{aligned}
-A(B_d^0 \rightarrow \rho^+ \rho^-) &= Tr + P, \\
-\sqrt{2}A(B_d^0 \rightarrow \rho^0 \rho^0) &= C - P.
\end{aligned} \tag{6}$$

Since a modification of P is involved, one sees immediately that f_T/f_L in $B^+ \rightarrow \rho^+ \rho^0$ will not be affected. This agrees with observation (see Table 2).

As detailed in the previous section, the extra SM contribution is taken into effect with the addition of a single amplitude, R :

$$\begin{aligned}
-A(B_d^0 \rightarrow \rho^+ \rho^-) &= Tr + P + R, \\
-\sqrt{2}A(B_d^0 \rightarrow \rho^0 \rho^0) &= C - P - R.
\end{aligned} \tag{7}$$

Tr , C , and P contribute mainly to the L polarization; the contributions to T arise at $O(1/m_b)$. In Ref [14], it was pointed out that C might contribute significantly to the transverse amplitude through the hard-spectator scattering for certain choice of the parameters representing this contribution. If one uses the default values of these parameters in Ref [14], C still contributes dominantly to the L polarization and so only R contributes to T . (The case where C also contributes to T is considered below.) It is understood that any contributions to T can be different for $T = +, -$, so that there are two new contributions, R_+ and R_- . (As noted earlier, in $B \rightarrow \phi K^*$, $A_+ \gg A_-$. If this were taken for $B \rightarrow \rho \rho$, we would have $R_+ \equiv R$, $R_- \simeq 0$.)

In order to estimate f_T/f_L for these decays, it is necessary to estimate the size of R_{\pm} . As discussed earlier, rescattering affects P_c , the c -quark contribution to the penguin amplitude. Thus, $|R_{\pm}| \sim |P|$. (A similar conclusion holds for $\bar{b} \rightarrow \bar{s}$

decays). The measured value of f_T/f_L in $B \rightarrow \phi K^*$ and $B \rightarrow \rho K^*$ is explained if $|R'| \sim |P'|$.) For penguin annihilation, the estimate is similar: $|R_\pm| \sim |P|$. Now, in Ref. [26], the relative sizes of the $B \rightarrow \rho\rho$ diagrams were roughly estimated as

$$1 : |Tr| \quad , \quad \mathcal{O}(\bar{\lambda}) : |C|, |P| \quad , \quad (8)$$

where $\bar{\lambda} \sim 0.2$. These estimates are expected to hold approximately in the SM. This shows that f_T/f_L is expected to be small in $B_d^0 \rightarrow \rho^+\rho^-$, since it is proportional to $(|R_+|^2 + |R_-|^2)/|Tr|^2 \sim |P|^2/|Tr|^2$. This agrees with observation (see Table 2).

However, f_T/f_L can be large in $B_d^0 \rightarrow \rho^0\rho^0$ since the contributions to the transverse and longitudinal polarizations are the same size. (However, see the estimate below.) It will be interesting to measure this precisely.

There are further tests. Since there is only one added amplitude, one has $|A_+(B_d^0 \rightarrow \rho^0\rho^0)| = |\bar{A}_-(\bar{B}_d^0 \rightarrow \rho^0\rho^0)|$, and similarly for A_- and \bar{A}_+ . If this is not found, penguin annihilation and rescattering will be ruled out.

We see that one can extract $|R_+|$ from $|A_+(B_d^0 \rightarrow \rho^0\rho^0)|$. As noted above, there are a variety of ways of obtaining $|R'_+|$ from $\bar{b} \rightarrow \bar{s}$ decays. One can then see if $|R_+|$ and $|R'_+|$ are related by flavor SU(3), thus testing penguin annihilation and rescattering. A similar exercise can be carried out for $|R_-|$ and $|R'_-|$. Note that the effect of SU(3) breaking must be included in the calculation. While we do not know its size, it should not be very large. A simple estimate of SU(3) breaking based on naive factorization confirms this as all the vector mesons (ρ , K^* , etc.) have masses and decay constants which are not very different.

In order to illustrate this, we use SU(3) to estimate f_T/f_L in $B_d^0 \rightarrow \rho^0\rho^0$ from $B^+ \rightarrow \rho^+K^{*0}$ decays. The transverse polarizations in these two modes are given by R and R' , respectively, where $R = \sqrt{|R_+|^2 + |R_-|^2}$, and similarly for R' . R and R' are related by SU(3): with penguin annihilation, $R = \mathcal{B}|V_{td}/V_{ts}|R'$, where \mathcal{B} is the measure of SU(3) breaking (with rescattering, the CKM ratio is $|V_{cd}/V_{cs}|$, which is of the same order as $|V_{td}/V_{ts}|$). In what follows, we neglect SU(3) breaking, so $\mathcal{B} = 1$. Now,

$$\begin{aligned} f_T/f_L(B_d^0 \rightarrow \rho^0\rho^0) &= |A_T(B_d^0 \rightarrow \rho^0\rho^0)|^2/|A_L(B_d^0 \rightarrow \rho^0\rho^0)|^2 \\ &= |V_{td}/V_{ts}|^2|A_T(B^+ \rightarrow \rho^+K^{*0})|^2/|A_L(B_d^0 \rightarrow \rho^0\rho^0)|^2 . \end{aligned} \quad (9)$$

Using experimental data, we find

$$\begin{aligned} |A_T(B^+ \rightarrow \rho^+K^{*0})|^2 &= (5.10 \pm 1.14) \times 10^{-16} \text{ GeV}^2 , \\ |A_L(B_d^0 \rightarrow \rho^0\rho^0)|^2 &= (2.10 \pm 0.81) \times 10^{-16} \text{ GeV}^2 , \end{aligned} \quad (10)$$

leading to

$$f_T/f_L(B_d^0 \rightarrow \rho^0\rho^0) = |V_{td}/V_{ts}|^2 (2.43 \pm 1.08) . \quad (11)$$

There are two points to be made here. The first is that this agrees with data taken directly from $B_d^0 \rightarrow \rho^0\rho^0$ (see Table 2):

$$f_T/f_L(B_d^0 \rightarrow \rho^0\rho^0) = (1 - f_L)/f_L = 0.16 \pm 0.15 . \quad (12)$$

Because of the large errors, the agreement is good, showing that there is no violation of SU(3). Equally, the measurement does not give a definite answer as to whether f_T/f_L is large or small. The second point is related to this: if central values are taken, f_T/f_L is not large after all. This shows that f_T/f_L is *not* guaranteed to be large in $B_d^0 \rightarrow \rho^0 \rho^0$. The reason for this is that, due to the additional amplitude C , f_L can be big, making f_T/f_L small. If one wishes to ensure a large f_T/f_L , it is better to use $\bar{b} \rightarrow \bar{d}$ modes which are dominated by one amplitude in the SM: P . This point will be used in the next section.

Finally, there is one more possible complication. Naively, the contribution from the diagram C to the transverse polarization is suppressed by $O(1/m_b)$. However, as already indicated earlier, in QCDf spectator corrections from the C diagram, which we denote as C_T , may contribute significantly to the transverse polarization [14]. If this is the case, there will be two contributions to the transverse polarization: R_i and C_T^i , $i = +, -$, with relative weak and strong phases. Assuming that the weak phase is taken from independent measurements, this leaves three parameters for a given transverse polarization, say $T = +$. One thus needs three pieces of information in $B_d^0 \rightarrow \rho^0 \rho^0$ to obtain these parameters.

Unfortunately, at present, this is not possible. This can be understood as follows. As above, $|A_+(B_d^0 \rightarrow \rho^0 \rho^0)|$ and $|\bar{A}_-(\bar{B}_d^0 \rightarrow \rho^0 \rho^0)|$ provide two measurements. The third piece of information would be to find the relative phase of these two amplitudes. Now, the angular analysis of $B \rightarrow V_1 V_2$ decays allows one to extract $\text{Im}(A_\perp A_0^*)$, $\text{Im}(A_\perp A_\parallel^*)$, and $\text{Re}(A_0 A_\parallel^*)$ [27]. This gives the relative phases of the A amplitudes. A similar exercise can be carried out for $\bar{B} \rightarrow \bar{V}_1 \bar{V}_2$, giving the relative phases of the \bar{A} amplitudes. Note that this does not give the relative phases of the A and \bar{A} amplitudes. However, for $\bar{b} \rightarrow \bar{s}$ decays, $A_0 = \bar{A}_0$. Thus, the angular analysis of both $B_d^0 \rightarrow \phi K^*$ and $\bar{B}_d^0 \rightarrow \phi \bar{K}^*$ *does* allow one to obtain the relative strong phases of all amplitudes. (This has been carried out, and is how ϕ_\perp and ϕ_\parallel of Table 1 were obtained in $B \rightarrow \phi K^*$.) But the same technique cannot be used for $\bar{b} \rightarrow \bar{d}$ decays, whose longitudinal polarization involves two amplitudes [Eq. (5)]. In order to obtain the relative strong phases of A and \bar{A} amplitudes in $\bar{b} \rightarrow \bar{d}$ decays, it will be necessary to perform a time-dependent angular analysis (this is described in detail in Sec. 5). This is possible, but it is a future measurement.

We therefore conclude that it is extremely difficult to perform the tests of penguin annihilation and rescattering described above if C_T^i contributions are present. The lesson here is that it is best to consider $\bar{b} \rightarrow \bar{d}$ decays for which f_T/f_L is expected to be large and which receive only one dominant contribution to the transverse polarization. We will return to this point in the next section.

4 U-Spin Pairs

In the past sections, we have stressed the idea of measuring f_T/f_L in $\bar{b} \rightarrow \bar{d}$ decays. But this raises the question: how does one choose the $\bar{b} \rightarrow \bar{d}$ decay to study? One

tool which is very useful in this regard is U-spin. U-spin is the symmetry that places d and s quarks on an equal footing, and is often given as transposing d and s quarks: $d \leftrightarrow s$. Pairs of B decays which are related by U-spin are given in Ref. [28]. In $B \rightarrow VV$ form, these are

1. $B_d^0 \rightarrow K^{*+} \rho^-$ and $B_s \rightarrow \rho^+ K^{*-}$,
2. $B_s \rightarrow K^{*+} K^{*-}$ and $B_d^0 \rightarrow \rho^+ \rho^-$,
3. $B_d^0 \rightarrow K^{*0} \rho^0$ and $B_s \rightarrow \bar{K}^{*0} \rho^0$,
4. $B^+ \rightarrow K^{*0} \rho^+$ and $B^+ \rightarrow \bar{K}^{*0} K^{*+}$,
5. $B_s \rightarrow K^{*0} \bar{K}^{*0}$ and $B_d^0 \rightarrow \bar{K}^{*0} K^{*0}$.

In all cases, the first decay is $\Delta S = 1$ ($\bar{b} \rightarrow \bar{s}$); the second is $\Delta S = 0$ ($\bar{b} \rightarrow \bar{d}$). Annihilation-type decays have been ignored. The procedure here is straightforward: one must measure the polarizations in the $\bar{b} \rightarrow \bar{s}$ decay, and compare them with the measurements in the corresponding $\bar{b} \rightarrow \bar{d}$ decay.

As noted previously, the best $\bar{b} \rightarrow \bar{d}$ decays are those for which f_T/f_L is expected to be large and which receive only one dominant contribution to the transverse polarization. Keeping only the largest contributions, the SM amplitudes for the $\Delta S = 0$ decays are

1. $A(B_s \rightarrow \rho^+ K^{*-}) = -[Tr + P]$,
2. $A(B_d^0 \rightarrow \rho^+ \rho^-) = -[Tr + P]$,
3. $\sqrt{2}A(B_s \rightarrow \bar{K}^{*0} \rho^0) = -[C - P]$,
4. $A(B^+ \rightarrow \bar{K}^{*0} K^{*+}) = P$,
5. $A(B_d^0 \rightarrow \bar{K}^{*0} K^{*0}) = P$.

The (potential) significant contributions to the transverse polarization + are

1. $B_s \rightarrow \rho^+ K^{*-} : Tr_T^+, R_+$,
2. $B_d^0 \rightarrow \rho^+ \rho^- : Tr_T^+, R_+$,
3. $B_s \rightarrow \bar{K}^{*0} \rho^0 : C_T^+, R_+$,
4. $B^+ \rightarrow \bar{K}^{*0} K^{*+} : R_+$,
5. $B_d^0 \rightarrow \bar{K}^{*0} K^{*0} : R_+$,

and similarly for $T = -$.

The first two decays are dominated by the tree diagram and are therefore expected to show small transverse polarization. This is reflected in the polarization measurement of $B_d^0 \rightarrow \rho^+ \rho^-$ (Table 2). The remaining three decays can have a large f_T/f_L .

In the past section, we have argued that it is best to consider $\bar{b} \rightarrow \bar{d}$ decays which receive only one dominant contribution to the transverse polarization, i.e. they are dominated by P in the SM. Given this, the best possibilities are the last two. We therefore consider (i) $B^+ \rightarrow K^{*0} \rho^+$ ($\bar{b} \rightarrow \bar{s}$) and $B^+ \rightarrow \bar{K}^{*0} K^{*+}$ ($\bar{b} \rightarrow \bar{d}$) and (ii) $B_s \rightarrow K^{*0} \bar{K}^{*0}$ ($\bar{b} \rightarrow \bar{s}$) and $B_d^0 \rightarrow \bar{K}^{*0} K^{*0}$ ($\bar{b} \rightarrow \bar{d}$). We urge the measurement of f_T/f_L in these pairs of decays.

The explanations of f_T/f_L in $B \rightarrow \phi K^*$ then make three predictions :

- f_T/f_L is expected to be large in both the $\bar{b} \rightarrow \bar{s}$ decay and the corresponding $\bar{b} \rightarrow \bar{d}$ decay.
- $|A_+|$ and $|\bar{A}_-|$ are expected to be equal in both the B and \bar{B} decays, and similarly for A_- and \bar{A}_+ .
- R'_i and R_i ($i = +, -$) can be extracted from the $\bar{b} \rightarrow \bar{s}$ and $\bar{b} \rightarrow \bar{d}$ decays, respectively. These should be related by flavor SU(3) (including SU(3) breaking).

If any of these predictions fail, penguin annihilation and rescattering are ruled out in the U-spin limit or for small U-spin breaking.

The ratio of f_T/f_L in these pairs of decays measures SU(3) breaking. For a given transverse polarization,

$$(f_T/f_L)_{\bar{b} \rightarrow \bar{d}} / (f_T/f_L)_{\bar{b} \rightarrow \bar{s}} = (|R_i|^2 / |R'_i|^2) / (|P_L|^2 / |P'_L|^2) , \quad (13)$$

where P_L and P'_L are the longitudinal parts of the penguin diagram in $\bar{b} \rightarrow \bar{d}$ and $\bar{b} \rightarrow \bar{s}$ transitions. Although one cannot prove it rigorously, it is likely that the SU(3) breaking in $(|R_i|/|R'_i|)$ is of the same size as that in $(|P_L|/|P'_L|)$, so that the net SU(3) breaking in this ratio is small. If one ignores SU(3) breaking for this reason, another prediction which can be used to test penguin annihilation and rescattering is that

$$(f_T/f_L)_{\bar{b} \rightarrow \bar{d}} = (f_T/f_L)_{\bar{b} \rightarrow \bar{s}} . \quad (14)$$

The breaking of SU(3) in the above equation is model dependent and, as indicated above, we expect to find small SU(3) breaking in Eq. 14 in models of penguin annihilation or rescattering. If it is found experimentally that the above relation is broken badly, then these models will have to invent a mechanism to generate large SU(3)-breaking effects or they will be ruled out. In other words, Eq. 14 can be used to constrain specific models of penguin annihilation and rescattering.

5 Distinguishing Penguin Annihilation and Rescattering

Up to now, we have not distinguished penguin annihilation and rescattering, arguing that their effects are very similar. However, is it possible to differentiate these two scenarios? As we will see in the present section, the answer is yes.

As noted above, rescattering involves only a change to P_c , while penguin annihilation involves only P_t . However, the weak phase of these pieces is different: $\phi(\text{rescattering}) \sim 0$, $\phi(\text{penguin annihilation}) \sim -\beta$. If this weak phase can be measured, one can distinguish penguin annihilation and rescattering.

This can be done as follows. Consider a penguin-dominated $\bar{b} \rightarrow \bar{d}$ decay in which the transverse polarization is observed to be large. Within the SM, this would be the result of a single dominant contribution originating from large rescattering or penguin annihilation. If the transverse amplitude in the penguin-dominated $\bar{b} \rightarrow \bar{d}$ decay is small then either rescattering or penguin annihilation is ruled out, or there is large SU(3) breaking in Eq. 14. Regardless, for small measured transverse polarization, we cannot assume the transverse amplitude to be dominated by a single contribution and so henceforth we will assume that a large transverse amplitude is observed in the penguin-dominated $\bar{b} \rightarrow \bar{d}$ decay. The transverse amplitude is then dominated by a single amplitude and we can parameterize this contribution as $Re^{i\phi}e^{i\delta}$, where ϕ and δ are the weak and strong phases, respectively. The transverse-polarization contribution to the CP-conjugate decay is then $Re^{-i\phi}e^{i\delta}$. Thus, the ratio of the transverse-polarization amplitude in the $\bar{b} \rightarrow \bar{d}$ and CP-conjugate decays is $e^{-2i\phi}$. In other words, this ratio measures the weak phase and allows us to distinguish penguin annihilation and rescattering.

In order to obtain this information, one needs to measure the relative phase of $A_T(\bar{b} \rightarrow \bar{d} \text{ decay})$ and $\bar{A}_T(b \rightarrow d \text{ decay})$. As discussed earlier, this can be obtained by performing a time-dependent angular analysis of the $\bar{b} \rightarrow \bar{d}$ decay. We give details of the procedure below.

Using CPT invariance, the full decay amplitudes for $B \rightarrow V_1 V_2$ can be written as [29, 30]

$$\begin{aligned}\mathcal{A} &= \text{Amp}(B \rightarrow V_1 V_2) = A_0 g_0 + A_{\parallel} g_{\parallel} + i A_{\perp} g_{\perp} , \\ \bar{\mathcal{A}} &= \text{Amp}(\bar{B} \rightarrow V_1 V_2) = \bar{A}_0 g_0 + \bar{A}_{\parallel} g_{\parallel} - i \bar{A}_{\perp} g_{\perp} ,\end{aligned}\tag{15}$$

where the g_{λ} are the coefficients of the helicity amplitudes written in the linear polarization basis. The g_{λ} depend only on the angles describing the kinematics [29].

Using the above equations, we can write the time-dependent decay rates as

$$\Gamma(\bar{B}(t) \rightarrow V_1 V_2) = e^{-\Gamma t} \sum_{\lambda \leq \sigma} \left(\Lambda_{\lambda\sigma} \pm \Sigma_{\lambda\sigma} \cos(\Delta M t) \mp \rho_{\lambda\sigma} \sin(\Delta M t) \right) g_{\lambda} g_{\sigma} . \tag{16}$$

Thus, by performing a time-dependent angular analysis of the decay $B(t) \rightarrow V_1 V_2$,

one can measure 18 observables (not all are independent). These are:

$$\begin{aligned}
\Lambda_{\lambda\lambda} &= \frac{1}{2}(|A_\lambda|^2 + |\bar{A}_\lambda|^2), & \Sigma_{\lambda\lambda} &= \frac{1}{2}(|A_\lambda|^2 - |\bar{A}_\lambda|^2), \\
\Lambda_{\perp i} &= -\text{Im}(A_\perp A_i^* - \bar{A}_\perp \bar{A}_i^*), & \Lambda_{\parallel 0} &= \text{Re}(A_\parallel A_0^* + \bar{A}_\parallel \bar{A}_0^*), \\
\Sigma_{\perp i} &= -\text{Im}(A_\perp A_i^* + \bar{A}_\perp \bar{A}_i^*), & \Sigma_{\parallel 0} &= \text{Re}(A_\parallel A_0^* - \bar{A}_\parallel \bar{A}_0^*), \\
\rho_{\perp i} &= \text{Re}(e^{-i\phi_M^q}[A_\perp^* \bar{A}_i + A_i^* \bar{A}_\perp]), & \rho_{\perp\perp} &= \text{Im}(e^{-i\phi_M^q} A_\perp^* \bar{A}_\perp), \\
\rho_{\parallel 0} &= -\text{Im}(e^{-i\phi_M^q}[A_\parallel^* \bar{A}_0 + A_0^* \bar{A}_\parallel]), & \rho_{ii} &= -\text{Im}(e^{-i\phi_M^q} A_i^* \bar{A}_i),
\end{aligned} \tag{17}$$

where $i = \{0, \parallel\}$ and ϕ_M^q is the weak phase factor associated with $B_q^0 - \bar{B}_q^0$ mixing. Note that the signs of the various $\rho_{\lambda\lambda}$ terms depend on the CP-parity of the various helicity states. We have chosen the sign of ρ_{ii} to be -1 , which corresponds to the final state ϕK^* . The quantities $\rho_{a,a}$, where $a = \parallel, \perp$, are sensitive to the weak phase between A_T and \bar{A}_T . Hence, for the case of penguin annihilation, the quantities $\rho_{a,a}/\Lambda_{a,a}$ are zero, while for rescattering these quantities are nonzero and equal $\pm \sin 2\beta$. (Note that since A_T is dominated by a single amplitude, we have $|A_a| = |\bar{A}_a|$.)

Which $\bar{b} \rightarrow \bar{d}$ decay should be used? It is necessary to consider one which receives only one dominant contribution to the transverse polarization and for which a time-dependent angular analysis can be done. Of the decays studied in the previous sections, there is only one which satisfies these requirements: $B_d^0 \rightarrow \bar{K}^{*0} K^{*0}$. Thus, the measurement of the time-dependent angular analysis here would allow one to distinguish penguin annihilation and rescattering, and we urge experimentalists to look at this. This time-dependent angular analysis can be performed with the all-charged-track final state $B_d^0 \rightarrow \bar{K}^{*0} K^{*0} \rightarrow K^- \pi^+ K^+ \pi^-$ without the need to reconstruct $K^{*0} \rightarrow K_S^0 \pi^0$ decays, which should facilitate experimental measurements once this decay is observed.

Finally, we note that even without the time-dependent analysis, the full angular analysis of $B \rightarrow \bar{K}^* K^*$ decays could help in distinguishing the models. If the strong phase difference between the $(P_c - P_u)$ and $(P_t - P_u)$ amplitudes in Eq. 5 is small, then the $\Delta\phi_{\perp, \parallel}$ parameters [2, 4] will be either negative or positive depending on the penguin annihilation or rescattering model. However, if the strong phase is not small, this will result in large direct CP violation in the A_0 amplitude which would be measured. A large strong phase would make it difficult to resolve ambiguities in $\Delta\phi_{\perp, \parallel}$, but limits on direct CP violation could constrain the strong phase difference. We also note that a similar method in $\bar{b} \rightarrow \bar{s}$ decays, such as $B \rightarrow \phi K^*$, does not work because both penguin annihilation and rescattering result in $\Delta\phi_{\perp, \parallel} = 0$, consistent with present data [4, 5, 7].

6 Conclusions

The final-state particles in $B \rightarrow \phi K^*$ are vector mesons (V), which means that this decay is in fact three separate decays, one for each polarization of the V (one longitudinal, two transverse). Naively, it is expected that the fraction of transverse decays, f_T , is much less than the fraction of longitudinal decays, f_L . However, it is observed that these fractions are roughly equal: $f_T/f_L \simeq 1$. This is the $B \rightarrow \phi K^*$ polarization puzzle.

Other, similar, polarization puzzles have been measured, all in $\bar{b} \rightarrow \bar{s}$ decays. Within the standard model, there have been several explanations of these results. However, if one requires a single explanation of all polarization puzzles, two possibilities remain: penguin annihilation and rescattering. Both of these also predict large f_T/f_L in certain $\bar{b} \rightarrow \bar{d}$ decays. Indeed, by looking at $\bar{b} \rightarrow \bar{d}$ decays, it is possible to test penguin annihilation and rescattering. This is the purpose of this paper.

We begin with $B \rightarrow \rho\rho$ decays. We show that f_T/f_L is expected to be small in $B^+ \rightarrow \rho^+\rho^0$ and $B_d^0 \rightarrow \rho^+\rho^-$; it is only in $B_d^0 \rightarrow \rho^0\rho^0$ that f_T/f_L can be large, though this is not guaranteed. Although penguin annihilation and rescattering are physically different, mathematically they are similar. If it is found that f_T/f_L is large in $B_d^0 \rightarrow \rho^0\rho^0$, it may be possible to test these explanations. For example, if one compares penguin annihilation or rescattering in $B_d^0 \rightarrow \rho^0\rho^0$ with that found in $\bar{b} \rightarrow \bar{s}$ decays, one can see if flavor SU(3) is respected.

Because large effects are not ensured in $B_d^0 \rightarrow \rho^0\rho^0$, it is useful to consider other $\bar{b} \rightarrow \bar{d}$ decays. We examine those which are related by U-spin to other $\bar{b} \rightarrow \bar{s}$ decays. We find two promising U-spin pairs: (i) $B^+ \rightarrow K^{*0}\rho^+$ ($\bar{b} \rightarrow \bar{s}$) and $B^+ \rightarrow \bar{K}^{*0}K^{*+}$ ($\bar{b} \rightarrow \bar{d}$) and (ii) $B_s \rightarrow K^{*0}\bar{K}^{*0}$ ($\bar{b} \rightarrow \bar{s}$) and $B_d^0 \rightarrow \bar{K}^{*0}K^{*0}$ ($\bar{b} \rightarrow \bar{d}$). A large f_T/f_L is predicted by penguin annihilation or rescattering in these decays. In addition, the measurement of f_T/f_L in these pairs of decays will allow us to test these explanations by seeing if flavor SU(3) is respected.

Up to now, we have treated penguin annihilation and rescattering as similar. However, it is possible to distinguish penguin annihilation from rescattering by performing a time-dependent angular analysis of $B_d^0 \rightarrow \bar{K}^{*0}K^{*0}$. This is difficult experimentally, but it may be possible at a future machine.

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